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"MATHEMATICAL ESTIMATION OF THE SYMMETRIC INPUT-OUTPUT MATRIX FROM THE ASSIMETRIC SUPPLY AND USE TABLES IN AN OPEN ECONOMY"

RODOLFO DE JESÚS HARO GARCÍA MA, PhD[1]

rodolfo.haro@inegi.gob.mx and rharo@prodigy.net.mx

Parque de Madrid No. 31, Col. Parques de la Herradura, Huixquilucan, Edo. de Mex. C. P. 52760, México

Abstract

The mathematical estimation of symmetric input-output tables (SIOT) from the compilation of the asymmetric supply and use tables at basic prices (SUTbp) based on information given to the national statistics offices by the economic units, is a crucial procedure due to non homogeneous production generated because contained by each industry secondary products are mixed with main or characteristic products. Due to this, to be useful for economic analysis, the estimation of the SIOT requires additional work, in such a way that the productive units integrated in each industry will be technologically homogeneous units of production.

This paper defines mathematically the underlying analytic structure of the asymmetric SUTbp and describes and formalizes mathematically the various methods proposed in the international norms and practices within this topic for the estimation of the SIOT from SUTbp. In short, it is proposed an integrated mathematical method to estimate the product-by-product and industry-by-industry SIOT in the case of an open economy including the imported intermediate consumption matrix, a theme confusedly treated in the international specialized manuals, particularly for the imports matrix treatment.

It is also shown that in the case of the methods not based on the production technology or the industry technology assumptions it is not possible to develop the symmetric imports table mathematically; thus, its estimation requires an adequate classification of industries and products than those designed and suggested by the United Nations (UN), like CPC, ISIC, etc., due that there is not an item by item correspondence vis-à-vis between the highest aggregation level of the ISIC and the highest disaggregated level of the CPC. This has obligated several countries (i.e. Canada, USA, Japan, European Union members, among others), to construct their own product classifications at the huge cost of loosing international comparability. In those cases the behavior has been a planning work in a manner that the imports estimation of the SIOT industry-by-industry could be by aggregation, which implies a correspondence without confusions between the product and industries classifications. Therefore, this major problem is still waiting for a solution by the Statistics Commission of the United Nations.

For the method based on the production technology, it is feasible to develop the symmetric imports table and fulfill the economic assumptions and structure of the complex theoretical basis of the input-output model, which is the foundation that justifies the high level of intellectual and technical efforts dedicated to estimate the product-by-product SIOT.

Key words: input-output analysis, supply and use tables, symmetric input-output table.

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[1] **Advisor to the President of the National Institute of Statistics and Geography of Mexico (Spanish acrostic: INEGI). The views expressed in this work are those of the author and do not necessarily represent those of INEGI´s policy. This document is published to elicit comments and to further debate. Javier Alejandro Barrientos y Olivares is responsible of the translation.**

Introduction

The central subject of this research is the development of a methodology for the mathematical derivation of the symmetric input-output table (SIOT) from the asymmetric supply-use tables at basic prices (SUTbp) for the case of an open economy. That is to say, from the *central framework of the System of National Accounts.*

The supply-use tables are two asymmetric matrices which describe the supply and use of goods and services produced and imported by a country in monetary units and represent the *central framework of the system of national accounts* as practiced by the more developed countries. It has been promoted by the United Nations through the publication of two current manuals: the System of National Accounts (United Nations, et al., 1993) and the Manual for the Compilation and Analysis of Input-Output Tables (United Nations, 2000) [2].

The use table is a double entry matrix on which to each industry is assigned a column vector and to each selected product a row vector. It constitutes an instrument of great importance for the organization and integration of a national system of economic statistics. Equally to other existing models for registering economic activity, it tries to obtain a wide vision of the production process. On this sense, this asymmetric input-output matrix registers the transactions of goods and services

[2] It is important to emphasize that the established international norms by European countries also provide a central source on this matter; especially Eurostat 1996 and Eurostat 2002.

elaborated by the economic agents that form the different industries in a country during a given period of time, generally one year.

A SIOT is a square intermediate cost matrix whose dimension could be product–by–product or industry–by–industry. In other words, a matrix of double entry on which to each selected industry (or product) is assigned a row vector and also a column vector[3]. On the row vector it is registered the destiny of the production according to user industry (or product). On the column vector production is measured according to the origin of its costs, whether for activities or products, depending from the selected symmetry type for the estimation of the SIOT.

The SIOT whose symmetry is industry-by-industry, shows on each element i,j the products of industry i used as inputs for the production of products of industry j. The table whose symmetry is product-by-product shows for each i,j element the product i used as input for the production of product j.

The background of this document is the group of principles established in chapters 2, 3 and 11 of (United Nations, 2000) and chapter 15 of (United Nations et. al., 1993).

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[3] On the field of mathematics, the hypothetic matrix U, would be symmetric if it were squared, as the case is and, simultaneously, $U_{ij} = U_{ji}$ for all i,j = 1…n. The symmetry is respect to the main diagonal and U would be also equal to her transposed U'. That is to say, a matrix is symmetric when it is equal to her transposed (the matrix U is symmetric if $U = U'$). The concept of symmetry on the input–output matrix is radically distinct; a matrix is symmetric when as much the rows as the columns use the same units. In the case of a product–by–product symmetric matrix is spoken about a symmetric matrix because each row i refers to the same product referred in column j when i = j; the concept of symmetry refers to this harmony on the position of the concepts from each column with the concepts of the corresponding row; harmony of concepts one respect to the other.

On section 1 the structure of the supply and use tables at basic prices (SUTbp) is introduced for the case of an open economy. Section 2 shows the set of equations and matrices that develop the analytical framework of the SUTbp necessary for the estimation of the SIOT on her different alternative structures.

The mathematical methods used to estimate the SIOT for an open economy from SUTbp are analyzed in section 3, in which it is developed the central matter of this research: ***the estimation of the SIOT from the mathematical structure of the accounting model of the SUTbp for an open economy.*** **Unprecedented contribution given the aggregated form - ambiguous and superficial – with which has been treated by the United Nations normative literature and in the documents from the international statistics offices, especially in treatment of the symmetric imports matrix, a case unadequately treated in the intersectoral analysis in the different documents elaborated by the international organizations; on those, foreign trade has been reduced to a simple vector of imports and an extra component of final demand: the exports vector. The target of this research is to analyze in detail several methodologies that are used for the calculation of the parallel accounting practices in distinct countries with the current international norms related with this theme as established by United Nations.**

Finally, a summary of the research, a set of conclusions and recommendations and bibliography is provided.

1 The production matrix structure at basic prices[4]

***Definition 1*. *Production matrix at basic prices*. The production matrix at basic prices, also called supply matrix of domestic products, describes the sources of the supply of products that constitute the domestic productive universe, at basic prices, of the economy for which it is constructed; it does not include trade and transport margins and does not have taxes net of subsidies on products, because the value of each cell is expressed at basic prices. In other words, products are valued deducting the trade and transport margins and the taxes net of subsidies on products. Therefore, in the columns are shown the industries and in the rows are described the products that are offered for those industries (see scheme 1).**

This matrix is part of the supply table and has pxa dimensions; in other words, the number of products by the quantity of industries determined for the specific case and according with the relative development of the economy or region under analysis[5]. It can be rectangular because the number of products is usually higher than the number of industries due to the nature of the economic units determined for the collection of basic data, even if they are establishments, given that on practice they produce not

[4] In this document the capital letters mean matrices, the small letters column vectors and the Greek letters scalars. Also, row vectors are represented as transposed column vectors and are represented with a prima symbol (´), as in the case of a transposed matrix too; i.e. the transposed vector a is shown as a´ and the transposed of matrix A is shown as A´. Vector and matrices dimensions can be equal only to the number of products (p) or to the number activities (a) and appear in the sub-indexes, whether alone or before the first comma, when the sub-indexes include other explanatory elements about the studied variable, especially in the measurement units; for the scalars generally its dimension is omitted.

[5] The production matrix $Q_{pxa,bp}$ (products-by-industries) is that part of the supply table that describes the domestic production.

only major products but also, on a less scale, a set of secondary products. Notice that on this context it is broken the acknowledged input-output theoretical model assumption that producers do not make any secondary production; in the theoretical model they produce their main and characteristic products only; consequently, the distinction between industries and products is worthless. Formally, the assumption that there is not joint production is broken. In other words, on each industry it is not obtained a homogeneous product that is distinct from that produced by the rest of industries.

Scheme 1 The production matrix structure

	Industries	Output of domestic products at basic prices
Products	$\mathbf{Q}_{pxa,bp}$	$\mathbf{q}_{p,bp}$
Total industry output at basic prices	$\mathbf{q}_{a,bp}{}'$	$\boldsymbol{\omega}_{bp,d}$

1.1 Production vector of domestic products ($\mathbf{q}_{p,bp}$)

***Definition 2**. The vector of domestic products at basic prices* **indicates that the production of each merchandise is equal to the sum of the quantities produced by each of the activities and it is equal to the sum of the corresponding row of the production matrix:**

$$\mathbf{q}_{p,bp} = \mathbf{Q}_{pxa,bp}\,\mathbf{u} \tag{1}$$

1.2 Vector of production of domestic industries ($\mathbf{q}_{a,bp}$)

Definition 3. ***The production vector at basic prices of the domestic industries*** **shows that the gross product of each industry is equal to the sum of the production of each merchandise or product produced by that activity or, by the same token, is equal to the sum of the corresponding column of the production matrix or, on an equivalent way, it is equal to the sum of the corresponding row of the transposed production matrix, as shown in the following equation:**

$$\mathbf{q}_{a,bp} = \mathbf{Q}_{pxa,bp}' \, \mathbf{u} \tag{2}$$

2 The analytical structure of the SUTpb in an open economy

On this section basic definitions are established plus the set of equations and matrices that constitute the analytical structure of the SUTbp.

2.1 Identities and production coefficients of the supply table

***Definition 4*. Matrix of output coefficients for industries by product**

$$\mathbf{D}_{pxa} = \mathbf{Q}_{pxa,bp}\ \mathrm{diag}(\mathbf{q}_{a,bp})^{-1} \qquad (3)$$

Where the matrix $(\mathrm{diag}(\mathbf{q}_{a,bp}))$ contains the vector $\mathbf{q}_{a,bp}$ in the main diagonal and zeros on the rest of the cells. Also, $[\mathrm{diag}(\mathbf{q}_{a,bp})]^{-1}$ is the inverse matrix of matrix $(\mathrm{diag}(\mathbf{q}_{a,bp}))$. The element i,j of a column of matrix $\mathbf{D}_{pxa}$ indicates the percentage of product i which was produced by industry j. The sum of columns is equal to one because for each industry is obtained 100% of the output produced.

***Definition 5.* Matrix of market shares or market quotas (Matrix of output coefficients for products by industry):**

$$\mathbf{D}_{axp} = \mathbf{Q}_{pxa,bp}{}'\ [\mathrm{diag}(\mathbf{q}_{p,bp}]^{-1} \qquad (4)$$

Where matrix (diag($\mathbf{q}_{p,bp}$)) contains vector $\mathbf{q}_{p,bp}$ in main diagonal and zeros on

the rest of the cells. Also, $[\mathrm{diag}(q_{p,bp})]^{-1}$ is the inverse matrix of the matrix $(\mathrm{diag}(q_{p,bp}))$. The element i,j of a column of matrix D_{axp} indicates the percentage of product i produced by industry j. The sum of the columns is equal to one because for each product is obtained 100% of its production, independently from the industry which has produced it. As may be noted by simple observation, after solving the transposed matrix of production it is obtained the following equation:

$$Q_{pxa,bp}' = D_{axp}\,\mathrm{diag}(q_{p,bp}) \tag{5}$$

2.2 The structure of the use table at basic prices for an open economy with integrated trade and transportation margins and the input-output coefficients.

2.2.1 General Structure

The valuation at basic prices of the use matrix for an open economy with integrated trade and transport margins of the scheme 2, signifies that the trade and transport margins have been subtracted from the price of the used products; These trade and transport margins are NOT included in the price of the used products. In other words, the trade and transport margins are shown in the rows corresponding to these products of the domestic and imported products matrices. Also, the corresponding amounts of taxes net of subsidies, that have been deducted also from the price of the used products, are shown in two separate matrices, one for the intermediate consumption and the other for final demand also with a vector of totals by product.

Therefore, all the calculations based in scheme 2 hold this structure.

Scheme 2 Use table at basic prices in an open economy with trade and transport margins included

	Industries	Final demand					Total products uses
		Exports f.o.b.	Household final expenditure	Government final expenditure	Gross capital formation	Total	
	(1)	(2)	(3)	(4)	(5)	(6)	(7) = (1)+…+(6)
Domestic products	$U_{pxa,bp,D}$	$x_{p,bp,D}$	$c_{p,bp,D}$	$g_{p,bp,D}$	$i_{p,bp,D}$	$f_{p,bp,D}$	$q_{p,bp,uD}$
Imported products	$U_{pxa,cif,M}$		$c_{p,cif,M}$	$g_{p,cif,M}$	$i_{p,cif,M}$	$f_{p,cif,M}$	$q_{p,cif,uM}$
Taxes less subsidies on products	$U_{pxa,tnp}$	$x_{p,tnp}$	$c_{p,tnp}$	$g_{p,tnp}$	$i_{p,tnp}$	$f_{p,bp,tnp}$	$t_{p,np}$
Direct purchases abroad by residents		ρ_{nr}	ρ_r			ρ_r	ρ_r
Direct purchases at home by non-residents	I		$-\rho_{nr}$	II		$-\rho_{nr}$	0
C.i.f./f.o.b. adjustment		α	$-\alpha$			0	0
Total uses at purchaser´s prices	$u_{a,pp,ci}{}'$	$\mu_{pp,x}$	$\mu_{pp,ch}$	$\mu_{pp,g}$	$\mu_{pp,i}$	μ_f	$\mu_{pp,t}$
Gross value added at basic prices	$v_{a,pb}{}'$						
Compensation of employees	$w_a{}'$						
Other taxes less subsidies on production	$t_{a,no}{}'$						
Consumption of fixed capital	$d_a{}'$ III			IV			
Operating surplus/Mixed incomes	$e_a{}'$						
Total industry output at basic prices	$q_{a,pb}{}'$						

***Definitions 6. Total use of products*. With the purpose to satisfy the fundamental equilibrium condition for an economic system: the equilibrium between supply of distinct domestic and imported products by a country or region should be equal to its demand or use, and also equal to the sum of the different components of use. In other words, total products use (total use of merchandises) ($\mathbf{q}_{p,bp,uT}$) is absorbed as intermediate product ($\mathbf{U}_{pxa,bp,T}\,\mathbf{u}$) or as final demand product ($\mathbf{f}_{p,bp,T}$)[6].**

$$\mathbf{q}_{p,bp,uT} = \mathbf{U}_{pxa,bp,T}\,\mathbf{u} + \mathbf{f}_{p,bp,T} \qquad (6)$$

Where u is a unitary column vector, $\mathbf{U}_{pxa,bp,T}$ is the total absorption matrix of products and $\mathbf{f}_{p,bp,T}$ is the total final demand vector of products.

The equilibrium between supply of distinct domestic products by a country or region should be equal to its demand or use, and also equal to the sum of the different components of use. In other words, total domestic products use (total use of domestic merchandises) ($\mathbf{q}_{p,bp,uD}$) is absorbed as intermediate product ($\mathbf{U}_{pxa,bp,D}\,\mathbf{u}$) or as final demand product ($\mathbf{f}_{p,bp,D}$).

6 This identity do not implies a generalized equilibrium in all products markets, it in fact includes the measurement of the disequilibrium as changes in inventories.

$$\mathbf{q}_{p,bp,uD} = \mathbf{U}_{pxa,bp,D}\,\mathbf{u} + \mathbf{f}_{p,bp,D} \qquad (7)$$

Where $\mathbf{U}_{pxa,bp,D}$ is the matrix of the value of the intermediate consumption of domestic products at basic prices by industry (absorption matrix of domestic products) and $\mathbf{f}_{p,bp,D}$ is the final demand vector of domestic products.

By the same token, total use of imported products (total use of each imported merchandise ($\mathbf{q}_{p,cif,uM}$) is absorbed as intermediate product ($\mathbf{U}_{pxa,cif,M}\ \mathbf{u}$) or as final demand product ($\mathbf{f}_{p,bp,M}$).

$$\mathbf{q}_{p,cif,uM} = \mathbf{U}_{pxa,cif,M}\,\mathbf{u} + \mathbf{f}_{p,bp,M} \qquad (8)$$

Where $\mathbf{U}_{pxa,bp,M}$ is the matrix of the value of the intermediate consumption of imported products to c.i.f. prices by industry (absorption matrix of imported products) and $\mathbf{f}_{p,bp,M}$ is the final demand vector of imported products.

2.2.2 Input-Output Coefficients Matrices (Technology Hypothesis).

2.2.2.1 Matrices of intermediate total, domestic and imported use coefficients of products per unit of industry production.

Definition 7. ***Matrix of intermediate total use coefficients of products at basic prices per unit produced by*** **industry (total intermediate products by unit produced per industry) is defined as[7]:**

$$\mathbf{B}_{\mathbf{pxa,T}} = \mathbf{U}_{\mathbf{pxa,bp,T}} \left[\mathrm{diag}(\mathbf{q}_{\mathbf{a,bp}})\right]^{-1} \quad \textbf{(9)}$$

The element i,j of a column of the matrix $\mathbf{B}_{\mathbf{pxa,T}}$, indicates the percentage of product i used as input by industry j.

Assumption 1. In the calculations to be introduced on the next section it will be assumed that in the matrix of intermediate total use coefficients of products at basic prices per unit produced by industry, the coefficients are proportional to the products of the industries where they enter.

Definition 8. ***The matrix of intermediate use coefficients of domestic products at basic prices per unit produced by industry*** **(intermediate domestic products by unit of production by industry) is defined as:**

$$\mathbf{B}_{\mathbf{pxa,D}} = \mathbf{U}_{\mathbf{pxa,bp,D}} \left[\mathrm{diag}(\mathbf{q}_{\mathbf{a,bp}})\right]^{-1} \quad \textbf{(10)}$$

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[7] **In this case:** $\mathbf{U}_{\mathbf{pxa,bp,T}} = \mathbf{U}_{\mathbf{pxa,bp,D}} + \mathbf{U}_{\mathbf{pxa,bp,M}}$

The element i,j of matrix $\mathbf{B}_{pxa,D}$ indicates the percentage on which the domestic product i was used as input by industry j.

Assumption 2. **In the calculations shown in the next section it is be assumed that in the matrix of intermediate use coefficients of domestic products at basic prices per unit produced by industry, the coefficients are proportional to the production of the activities where they enter.**

Definition 9. The matrix of intermediate use coefficients of imported products at basic prices (c.i.f.) per unit produced by industry **(imported intermediate products per unit produced by industry) is defined as:**

$$\mathbf{B}_{pxa,M} = \mathbf{U}_{pxa,bp,M} \left[\mathrm{diag}(\mathbf{q}_{a,bp})\right]^{-1} \tag{11}$$

The element i,j of matrix $\mathbf{B}_{pxa,M}$ indicates the percentage on which the imported product i was used as input by industry j.

Assumption 3. **In the calculations shown in the next section it is assumed that in the matrix of intermediate use coefficients of imported products at basic prices (c.i.f.) per unit produced by industry, the coefficients are proportional to the production of the activities where they enter.**

2.2.2.2 Matrix of Coefficients of Value Added by Category of Value Added per unit produced by industry

***Definition 10.** The Matrix of Coefficients of Value Added by Category of Value Added per unit produced by industry* **is defined as:**

$$\mathbf{L}_{vxa} = \mathbf{V}_{vxa} \; [\mathrm{diag}(\mathbf{q}_{a,bp})]^{-1} \tag{12}$$

The element i,j from matrix $\mathbf{L}_{vxa}$ indicates the percentage on which the value added category i was used as input by industry j.

Assumption 4. **In the calculations shown in the next section it is assumed that in the matrix of coefficients of value added by category of value added per unit produced by industry, the coefficients are proportional to the production of the industries where they enter.**

2.2.2.3 Vector of Coefficients of Total Value Added per unit produced by the industries.

***Definition 11.** The vector of coefficients of the value added by category of value added per unit produced by the industries* **is defined as:**

$$\mathbf{u}'\mathbf{L}_{vxa} = \mathbf{u}' \, \mathbf{V}_{vxa} \; [\mathrm{diag}(\mathbf{q}_{a,bp})]^{-1} = \mathbf{v}_a' \, [\mathrm{diag}(\mathbf{q}_{a,bp})]^{-1} \tag{13}$$

The element j of vector $\mathbf{u}'\mathbf{L}_{vxa}$ indicates the percentage on which the total value added was generated by industry j.

Assumption 5. **In the calculations shown in the next section it is assumed that in the vector of coefficients of value added by category of value added per unit produced by the industries the coefficients are proportional to the production of the industries where they are used.**

2.2.3 Estimation of the matrices of production and use with the technological and production assumptions and the *fundamental asymmetric analytical equation* of the input-output analysis.

Finally, it can be identified a set of equations that will be useful for understanding the input-output system, especially for estimating SIOT and when designing input-output analysis models.

2.2.3.1 Matrix of Total Intermediate Use

The intermediate use matrices could be calculated solving the definitions of the matrices of use coefficients by unit produced per industry just described; that is:

$$\mathbf{U}_{pxa,bp,T} = \mathbf{B}_{pxa,T}\ \mathrm{diag}(\mathbf{q}_{a,bp}) \tag{14}$$

$$\mathbf{U}_{pxa,bp,D} = \mathbf{B}_{pxa,D}\ \mathrm{diag}(\mathbf{q}_{a,bp}) \tag{15}$$

$$\mathbf{U}_{pxa,cif,M} = \mathbf{B}_{pxa,M}\ \mathrm{diag}(\mathbf{q}_{a,bp}) \tag{16}$$

The element i,j of these matrices indicates, respectivelly, the input i, total, domestic or imported, which was used by industry j.

2.2.3.2 Matrix of Production (each industry produces merchandises in their own fixed proportions)

The production matrix, as well, could be calculated solving the definition of the matrix of output coefficients for industries by product, that is:

$$\mathbf{Q}_{pxa,bp} = \mathbf{D}_{pxa}\ \mathrm{diag}(\mathbf{q}_{a,bp}) \tag{17}$$

2.2.3.3 Transposed Matrix of Production (implies that the merchandises are originated in fixed proportions within the diverse industries)

As explained from the beginning of this section, the transposed matrix of production, could be calculated solving the definition of the matrix of market share or matrix of market quotas; that is:

$$\mathbf{Q}_{pxa,bp}{}' = \mathbf{D}_{axp}\ \mathrm{diag}(\mathbf{q}_{p,bp}) \tag{18}$$

Implies that the merchandises are originated in fixed proportions within the diverse industries.

The asymmetric analytical equations fundamental for input-output analysis.

Substituting 14 into 6 we obtain the *total asymmetric analytical equation fundamental* for input-output analysis derived directly from the SUTbp:

$$\mathbf{q}_{p,bp,uT} = \mathbf{B}_{pxa,T}\ \mathbf{q}_{a,bp} + \mathbf{F}_{pxf,T}\ \mathbf{u} \tag{19}$$

Substituting 15 into 6 we obtain the *domestic products asymmetric analytical*

equation fundamental **for input-output analysis:**

$$\mathbf{q_{p,bp,uD} = B_{pxa,D}\, q_{a,bp} + F_{pxf,D}\, u} \qquad \textbf{(20)}$$

Substituting 16 into 6 we obtain the ***imported products asymmetric analytical equation fundamental*** **for input-output analysis:**

$$\mathbf{q_{p,cif,uM} = B_{pxa,M}\, q_{a,bp} + F_{pxf,M}\, u} \qquad \textbf{(21)}$$

(In this case: $\mathbf{q_{p,bp,uT} = q_{p,bp,uD} + q_{p,cif,uM}}$**).**

3 The transformation of the supply and use tables (SUTbp) to the input-output matrix: an open economy case

The structure of the SUTbp introduced in Scheme 2 is the starting point to estimate the SIOT at basic prices whose structure is contained in Schemes 3 and 4. Also, the set of definitions and assumptions, equations and matrices that form the analytical structure of the SUTbp introduced in the previous section, especially the technology hypothesis (input-output coefficients) and the hypothesis of the productive structure (production coefficients), are used to show the mathematical methods defined to estimate the SIOT from the SUTbp.

The secondary residual products that appear in the SUTbp after applying statistical methods, especially the method of transfer of products and inputs or the method of redefinition, are the generators of the characteristic heterogeneity of the asymmetric tables. In this document are studied the mathematical methods used for the estimation of the SIOT[8].

On this research it is introduced a different terminology compared with that used in the Manual (United Nations, 2000) due to the approach of recent studies it has been found that the underlying assumptions from different methods not only are related with product technology and industry technology assumptions, but also with the fixed products sales structure assumption and the fixed industry sales structure

8 For an exhaustive review about the approaches for the treatment of secondary products see: Rueda Cantuche José Manuel (2004). This reference also includes an extensive bibliography about the subject matter of this research.

Scheme 3 Industry by industry SIOT at basic prices in an open economy with trade and transport margins included

	Industries	Final demand					Total use of products
		Exports f.o.b.	Household final expenditure	Government final expenditure	Gross capital formation	Total	
	(1)	(2)	(3)	(4)	(5)	(6)	(7) = (1) + … + (6)
Domestic industries	$U_{axa,bp,D}$	$x_{a,bp,D}$	$c_{a,bp,D}$	$g_{a,bp,D}$	$i_{a,bp,D}$	$f_{a,bp,D}$	$q_{a,bp,uD}$
Foreign industries	$U_{axa,cif,M}$		$c_{a,cif,M}$	$g_{a,cif,M}$	$i_{a,cif,M}$	$f_{a,bp,M}$	$q_{a,cif,uM}$
Taxes less subsidies on products	$U_{axa,tnp}$	$x_{a,tnp}$	$c_{a,tnp}$	$g_{a,tnp}$	$i_{a,tnp}$	$f_{a,bp,tnp}$	$t_{a,np}$
Direct purchases abroad by residents		ρ_{nr}	ρ_r			ρ_r	ρ_r
Direct purchases at home by non-residents	I		$-\rho_{nr}$	II		$-\rho_{nr}$	0
C.i.f./f.o.b. adjustment		α	$-\alpha$			0	0
Total uses at purchaser´s prices	$u_{a,pp,ci}'$	$\mu_{pp,x}$	$\mu_{pp,ch}$	$\mu_{pp,g}$	$\mu_{pp,i}$	μ_f	$\mu_{pp,t}$
Gross value added at basic prices	$v_{a,pb}'$						
Compensation of employees	w_a'						
Other taxes less subsidies on production	$t_{a,no}'$						
Consumption of fixed capital	d_a III			IV			
Operating surplus/Mixed incomes	e_a'						
Total industry output at basic prices	$q_{a,pb}'$						

Scheme 4 Product by product SIOT at basic prices in an open economy with trade and transport margins included

	Products	Final demand					Total use of products
		Exports f.o.b.	Household final expenditure	Government final expenditure	Gross capital formation	Total	
	(1)	(2)	(3)	(4)	(5)	(6)	(7) = (1) + … + (6)
Domestic products	$U_{pxp,bp,D}$	$x_{p,bp,D}$	$c_{p,bp,D}$	$g_{p,bp,D}$	$i_{p,bp,D}$	$f_{p,bp,D}$	$q_{p,bp,uD}$
Imported products	$U_{pxp,cif,M}$		$c_{p,cif,M}$	$g_{p,cif,M}$	$i_{p,cif,M}$	$f_{p,bp,M}$	$q_{p,cif,uM}$
Taxes less subsidies on products	$U_{pxp,tnp}$	$x_{p,tnp}$	$c_{p,tnp}$	$g_{p,tnp}$	$i_{p,tnp}$	$f_{p,bp,tnp}$	$t_{p,np}$
Direct purchases abroad by residents		ρ_{nr}	ρ_r			ρ_r	ρ_r
Direct purchases at home by non-residents			$-\rho_{nr}$			$-\rho_{nr}$	0
C.i.f./f.o.b. adjustment		α	$-\alpha$			0	0
Total uses at purchaser´s prices	$u_{p,pp,ci}{}'$	$\mu_{pp,x}$	$\mu_{pp,ch}$	$\mu_{pp,g}$	$\mu_{pp,i}$	μ_f	$\mu_{pp,t}$
Gross value added at basic prices	$v_{p,bp}{}'$						
Compensation of employees	$w_p{}'$						
Other taxes less subsidies on production	$t_{p,no}{}'$						
Consumption of fixed capital	$d_p{}'$						
Operating surplus/Mixed incomes	$e_p{}'$						
Total industry output at basic prices	$q_{p,bp}{}'$						

assumption, as established in the next paragraph:

"It has been pointed out that the terminology first introduced in the 1968 SNA is misleading, when the term "technology" is used also in connection with the construction of a SIOT of the industry-by-industry type from supply and use tables (SUT). An overview of the revised terminology used in this paper is shown in chart 2. The main distinction is not between two technology assumptions, but between technology assumptions on the one hand, and sales structure assumptions on the other". (Thage, Bent, 2005). Look also: (Konijn P.A. and Steenge A.E., 1995).

As it is shown in the following paragraph, the former has been recently acknowledged by OECD, when referring to one of the mathematical methods that I will mention:

"But as Thage 2005 shows this description of the conversion (industry-by-technology assumption) is inaccurate where industry-by-industry tables are concerned, and, is better described as a fixed product sales structure assumption. In other words the conversion merely assumes that the proportion of domestically produced commodity A bought by industry B from industry C is proportional to industry C´s share of the total (domestic) economy production of commodity A. Put this way, it is clear that this is a far less demanding assumption than that implied by the equivalent, but differently named, industry technology assumption". (Norihiko Yamano and Nadim Ahmad, 2006).

These four mathematical methods proposed for the estimation of the SIOT from the asymmetrical and heterogeneous SUTbp are analyzed for an open economy. First, the relevant matrices are derived for the segment of domestic products and then for the segment of foreign products considering that an imported SIOT is looked for. For pure mathematical reasons from here on it is assumed that the use table – and thus, the production matrix – is square and the cases when this condition is absolutely necessary are identified.

3.1 The mathematical methods for domestic products segment[9].

3.1.1 The asymmetric analytical equation.

As noted in the previous section, substituting 15 into 7 we obtain the *fundamental asymmetric analytical equation* of the segment of domestic products and industries of the input–output analysis obtained directly from the SUTbp:

$$\mathbf{q}_{p,bp,uD} = \mathbf{B}_{pxa,D}\, \mathrm{diag}(\mathbf{q}_{a,bp})\, \mathbf{u} + \mathbf{F}_{pxf,D}\, \mathbf{u} = \mathbf{U}_{pxa,bp,D} \left[\mathrm{diag}(\mathbf{q}_{a,bp})\right]^{-1} \mathbf{q}_{a,bp} + \mathbf{f}_{p,bp,D} \qquad (22)$$

An other form:

$$\boxed{\mathbf{q}_{p,bp,uD} = \mathbf{B}_{pxa,D}\, \mathbf{q}_{a,bp} + \mathbf{f}_{p,bp,D}} \qquad (23)$$

The vector of total use of domestic products with trade and transport margins included is a linear function of the production of industries $\mathbf{q}_{p,bp,uD} = \mathbf{f}(\mathbf{q}_{a,bp})$.

But, this equation has no solution because generally $\mathbf{q}_{p,bp,uD} \neq \mathbf{q}_{a,bp}$ even in case that the number of products be equal to the number of industries. In other words, this analytical equation involves an intermediate utilization coefficients matrix of domestic products with trade and transport margins included per unit produced by industry ($\mathbf{B}_{pxa,D}$), which is asymmetrical because in the rows are used different units than

[9] The mathematical demonstrations of section 3 come from (Haro García, R. de J., 2006).

those used in the columns (outputs in rows and industries in columns). Also because the matrix of total intermediate use, defined in the previous section, is also asymmetric by the same reasons. It is equal to:

$$\mathbf{U}_{pxa,bp,D} = \mathbf{B}_{pxa,D}\ \mathrm{diag}(\mathbf{q}_{a,bp}) = \mathbf{U}_{pxa,bp,D}\ [\mathrm{diag}(\mathbf{q}_{a,bp})]^{-1}\ \mathrm{diag}(\mathbf{q}_{a,bp}) \tag{24}$$

3.1.2 Product-by-product SIOT of technical coefficients for the segment of domestic products based on production or commodity technology assumption.

We solve diag($\mathbf{q}_{a,bp}$) from 17 and obtain:

$$\mathrm{diag}(\mathbf{q}_{a,bp}) = (\mathbf{D}_{pxa})^{-1}\ \mathbf{Q}_{pxa,bp} \tag{25}$$

Then, we substitute 25 in 23 to solve the system:

$$\mathbf{q}_{p,bp,uD} = \mathbf{B}_{pxa,D}\ (\mathbf{D}_{pxa})^{-1}\mathbf{Q}_{pxa,bp}\ \mathbf{u} + \mathbf{F}_{pxf,D}\ \mathbf{u} \tag{26}$$

We substitute equation 1 in 26 and obtain the *fundamental analytical equation of the domestic products based on production or commodity technology assumption*:

$$\boxed{\mathbf{q}_{p,bp,uD} = \mathbf{B}_{pxa,D}\ (\mathbf{D}_{pxa})^{-1}\ \mathbf{q}_{p,bp,uD} + \mathbf{F}_{pxf,D}\ \mathbf{u}} \tag{27}$$

This equation is a system of linear equations on which the vector of total use of domestic products with trade and transport margins included is a linear function of itself $\mathbf{q}_{p,bp,uD} = f(\mathbf{q}_{p,bp,uD})$.

We obtain the necessary results to estimate the *product-by-product SIOT of intermediate domestic consumption based on production or commodity technology assumption as a function of the original variables of the asymmetric SUTbp which is*:

$$\mathbf{U_{P,pxp,bp,D} = B_{pxa,D}\,(D_{pxa})^{-1}\,diag(q_{p,bp,uD}) = U_{pxa,bp,D}\,[diag(q_{a,bp})]^{-1}\,[diag(q_{a,bp})]Q_{pxa,bp}{}^{-1}diag(q_{p,bp,uD}) = U_{pxa,bp,D}\,Q_{pxa,bp}{}^{-1}\,diag(q_{p,bp,uD})} \quad \textbf{(28)}$$

Thus, the *SIOT of intermediate domestic consumption* is symmetric given that in the rows are used the same units as in the columns (products in rows and columns).

Thus, the *product-by-product SIOT of technical coefficients for the segment of domestic products based on production or commodity technology assumption is*:

$$\mathbf{A_{P,pxp,bp,D} = B_{pxa,D}\,(D_{pxa})^{-1} = U_{pxa,bp,D}\,[diag(q_{a,bp})]^{-1}diag(q_{a,bp})\,Q_{pxa,bp}{}^{-1}} \quad \textbf{(29)}$$

And by the definition of the matrix of coefficients of distribution of the production of industries per product:

$$\mathbf{D_{pxa} = Q_{pxa,bp}\,diag(q_{a,bp})^{-1}} \quad \textbf{(30)}$$

We know that:

$$\mathbf{(D_{pxa})^{-1} = diag(q_{a,bp})\,(Q_{pxa,bp})^{-1}} \quad \textbf{(31)}$$

Thus, the *fundamental analytical equation of the domestic products based on production or commodity technology assumption* can be solved *in the case that the SUTbp be squared*. The system solution is:

$$q_{p,bp,uD} = [I - B_{pxa,D} [D_{pxa}]^{-1}]^{-1} F_{pxf,D} u =$$
$$[I - B_{pxa,D} (diag(q_{a,bp})) (Q_{pxa,bp})^{-1}]^{-1} F_{pxf,D} u \quad (32)$$

Where I represents the identity matrix.

Notice that equation 32 implies the presence of negative numbers in both the product-by-product SIOT of technical coefficients for the segment of domestic products based on production or commodity technology assumption and in the product-by-product SIOT of intermediate use for the segment of domestic products based on production or commodity technology assumption.

The Value Added matrix by product based in the product technology assumption ($V_{P,vxp}$)

Resulting from equation 27 and the estimation of the *product-by-product SIOT of technical coefficients* based on production or commodity technology assumption from equation 29 we obtain:

$$q_{p,bp,uD} = A_{P,pp,bp,D}\ q_{p,bp,uD} + F_{pxf,D}\ u = U_{P,pxp,bp,D}\ u + F_{pxf,D}\ u \quad (33)$$

Thus, it is deducted that the pre-multiplication of the domestic products vector by the product-by-product SIOT of technical coefficients for the segment of domestic products based on the product technology assumption is equal to the sum of the rows of the symmetric product-by-product SIOT of intermediate use for the segment of domestic products based on production or commodity technology assumption:

$$\mathbf{A}_{P,pxp,bp,D}\ \mathbf{q}_{p,bp,uD} = \mathbf{U}_{P,pxp,bp,D}\ \mathbf{u} \qquad (34)$$

Considering that:

$$\mathbf{u} = \mathrm{diag}(\mathbf{q}_{p,bp,uD})^{-1}\ \mathbf{q}_{p,bp} \qquad (35)$$

Then:

$$\mathbf{A}_{P,pxp,bp,D}\ \mathbf{q}_{p,bp,uD} = \mathbf{U}_{P,pxp,bp,D}\ \mathrm{diag}(\mathbf{q}_{p,bp,uD})^{-1}\ \mathbf{q}_{p,bp,uD} \qquad (36)$$

In other words:

$$\mathbf{B}_{pxa,D}\ [\mathbf{D}_{pxa}]^{-1}\ \mathbf{q}_{p,bp,uD} = \mathbf{U}_{P,pxp,bp,D}\ \mathrm{diag}(\mathbf{q}_{p,bp,uD})^{-1}\ \mathbf{q}_{p,bp,uD} \qquad (37)$$

Therefore:

$$\mathbf{B}_{pxa,D}\ [\mathbf{D}_{pxa}]^{-1}\ \mathbf{q}_{p,bp,uD}\ \mathbf{q}_{p,bp,uD}' = \mathbf{U}_{P,pxp,bp,D}\ \mathrm{diag}(\mathbf{q}_{p,bp,uD})^{-1}\mathbf{q}_{p,bp,uD}\ \mathbf{q}_{p,bp,uD}' \qquad (38)$$

And:

$$\mathbf{B}_{pxa,D}\ [\mathbf{D}_{pxa}]^{-1}\ (\mathbf{q}_{p,bp,uD}\ \mathbf{q}_{p,bp,uD}')(\mathbf{q}_{p,bp,uD}\ \mathbf{q}_{p,bp,uD}')^{-1} = \mathbf{U}_{P,pxp,bp,D}\ \mathrm{diag}(\mathbf{q}_{p,bp,uD})^{-1} \qquad (39)$$

Then:

$$\mathbf{B}_{pxa,D}\ [\mathbf{D}_{pxa}]^{-1} = \mathbf{U}_{P,pxp,bp,D}\ \mathrm{diag}(\mathbf{q}_{p,bp,uD})^{-1} \qquad (40)$$

Thus:

$$U_{P,pxp,bp,D} = B_{pxa,D}\,[D_{pxa}]^{-1}\,diag(q_{p,bp,uD}) = U_{pxa,bp,D}\,Q_{pxa,bp}^{-1}\,diag(q_{p,bp,uD}) \quad (41)$$

Afterwards, taking definitions from $B_{pxa,bp,D}$ and $(D_{pxa})^{-1}$:

$$U_{P,pxp,bp,D} = U_{pxa,bp,D}\,diag(q_{a,bp})^{-1}\,diag(q_{a,bp})\,(Q_{pxa,bp})^{-1} diag(q_{p,bp,uD}) \quad (42)$$

Which can be simplified, as established in equations 28 and 41, by the following manner:

$$\boxed{U_{P,pxp,bp,D} = U_{pxa,bp,D}\ (Q_{pxa,bp})^{-1}\ diag(q_{p,bp,uD})} \quad (43)$$

From which we obtain the transformation matrix for the columns of the value added of dimensions vxa:

$$\boxed{[Q_{pxa,bp}]^{-1}\ diag(q_{p,bp,uD})} \quad (44)$$

In other words, the *value added matrix based on production or commodity technology assumption* of dimensions categories of value added by product (vxp) would be:

$$\boxed{V_{P,vxp} = V_{vxa}\ (Q_{pxa,bp})^{-1}\ diag(q_{p,bp})} \quad (45)$$

Also, the *value added vector by product based on production or commodity technology assumption* would be:

$$\boxed{\mathbf{v}_{P,p,bp}{}' = \mathbf{u}'\mathbf{V}_{P,vxp} = \mathbf{u}'\mathbf{V}_{vxa}\,(\mathbf{Q}_{pxa,bp,})^{-1}\,\mathrm{diag}(\mathbf{q}_{p,bp})} \quad (46)$$

3.1.3 Industry-by-industry SIOT of technical coefficients for the segment of domestic products based on fixed industry sales structure assumption.

Substituting 17 into 1 and considering that the vector of the supply value of the domestic products at basic prices by product ($\mathbf{q}_{p,bp}$) is equal to the vector of the value of total use of domestic products with trade and transport margins included ($\mathbf{q}_{p,bp,uD}$), we obtain:

$$\mathbf{q}_{p,bp} = \mathbf{q}_{p,bp,uD} = \mathbf{D}_{pxa}\,\mathbf{q}_{a,bp} \quad (47)$$

Which is sutituted in the *fundamental asymmetric analytical equation for the segment of domestic products* of input-output analysis derived directly from SUTbp (equation 22):

$$\mathbf{D}_{pxa}\,\mathbf{q}_{a,bp} = \mathbf{B}_{pxa,D}\,\mathbf{q}_{a,bp} + \mathbf{F}_{pxf,D}\,\mathbf{u} \quad (48)$$

Thus, we obtain the *fundamental analytical equation industry-by-industry for the segment of domestic products based on fixed industry sales structure assumption* derived mathematically from the SUTbp.

$$\boxed{\mathbf{q}_{a,bp} = (\mathbf{D}_{pxa})^{-1}\,\mathbf{B}_{pxa,D}\,\mathbf{q}_{a,bp} + (\mathbf{D}_{pxa})^{-1}\,\mathbf{F}_{pxf,D}\,\mathbf{u}} \quad (49)$$

This equation is a system of linear equations on which the vector of production of the industries is a linear function of itself $q_{a,bp} = f(q_{a,bp})$.

Thus, the *<u>industry-by-industry SIOT of technical coefficients for the segment of the domestic products based on fixed industry sales structure assumption</u>* is:

$$\boxed{A_{FIS,axa,bp,D} = (D_{pxa})^{-1} B_{pxa,D} =} \quad \boxed{\mathrm{diag}(q_{a,bp})\ Q_{pxa,bp}{}^{-1}\ U_{pxa,bp,D}\ [\mathrm{diag}(q_{a,bp})]^{-1}} \tag{50}$$

Thus, the *<u>fundamental input-output analytical equation for the segment of domestic products based on fixed industry sales structure assumption (equation 49)</u>* can be solved just *in case that the SUTbp be square only*. The system solution is:

$$\boxed{q_{a,bp} = (I- (D_{pxa})^{-1} B_{pxa,D})^{-1} (D_{pxa})^{-1} F_{pxf,D}\ u} \tag{51}$$

Therefore, we obtain the correct results to estimate the *<u>industry-by-industry intermediate consumption SIOT for the segment of domestic products based on the fixed industry sales structure assumption</u>*:

$$\begin{gathered} U_{FIS,axa,bp,D} = (D_{pxa})^{-1} B_{pxa,D}\ \mathrm{diag}(q_{a,bp}) = \\ [\mathrm{diag}(q_{a,bp})]\ Q_{pxa,bp}{}^{-1}\ U_{pxa,bp,D}\ [\mathrm{diag}(q_{a,bp})]^{-1}\ \mathrm{diag}(q_{a,bp}) = \\ [\mathrm{diag}(q_{a,bp})]\ Q_{pxa,bp}{}^{-1}\ U_{pxa,bp,D} \end{gathered} \tag{52}$$

Thus, the intermediate use matrix is symmetric since in the rows are used the same units than in the columns (*industries* in rows and columns).

The domestic final demand matrix by industries based on the fixed sales structure assumption of the industries ($\mathbf{F}_{\mathbf{FIS,axf,bp,D}}$)

From 49 we observe that the product final demand has been transformed into industries according with the following equation:

$$(\mathbf{D}_{\mathbf{pxa}})^{-1}\ \mathbf{F}_{\mathbf{pxf,D}} \tag{53}$$

Consequently, we identify the transformation matrix for the rows of the final demand matrix with dimensions pxf:

$$(\mathbf{D}_{\mathbf{pxa}})^{-1} \tag{54}$$

In other words, the *<u>domestic final demand matrix industry-by-category (axf) of final demand based on fixed industry sales structure assumption</u>* would be:

$$\mathbf{F}_{\mathbf{FIS,axf,bp,D}} = (\mathbf{D}_{\mathbf{pxa}})^{-1}\ \mathbf{F}_{\mathbf{pxf,D}} = [\mathrm{diag}(\mathbf{q}_{\mathbf{a,bp}})]\ \mathbf{Q}_{\mathbf{pxa,bp}}^{-1}\ \mathbf{F}_{\mathbf{pxf,D}} \tag{55}$$

Also, the *<u>final demand value vector by industry based on fixed industry sales structure assumption</u>* for the domestic products would be:

$$\boxed{\mathbf{f}_{\mathbf{FIS,a,bp,D}}\ \mathbf{u} = (\mathbf{D}_{\mathbf{pxa}})^{-1}\ \mathbf{F}_{\mathbf{pxf,D}}\ \mathbf{u}} \tag{56}$$

3.1.4 Product-by-product SIOT of technical coefficients for the segment of domestic products based on industry technology assumption.

In 2 it is substituted $Q_{pxa,bp}{}'$ by 18 and we obtain:

$$q_{a,bp} = D_{axp}\ \mathrm{diag}(q_{p,bp})\ u = D_{axp}\ \mathrm{diag}(q_{p,bp,uD})\ u \qquad (57)$$

Then, this equation is substituted by $q_{a,bp}$ in the *fundamental asymmetric analytical equation* to obtain the *analytical equation for the segment of domestic products based on industry technology assumption* and mathematically derived from the SUTbp:

$$\boxed{q_{p,bp,uD} = B_{pxa,D}\ D_{axp}\ q_{p,bp,uD} + F_{pxf,D}\ u} \qquad (58)$$

This equation is also a system of linear equations on which the total use value vector of domestic products with trade and transport margins included is a linear function of itself $q_{p,bp,uD} = f(q_{p,bp,uD})$.

Therefore, the *product-by-product SIOT of technical coefficients for the segment of domestic products based on industry technology assumption* is equal to:

$$\boxed{A_{A,pxp,bp,D} = B_{pxa\ ,D}\ D_{axp} =}$$
$$\boxed{U_{pxa,bp,D}\ [\mathrm{diag}(q_{a,bp})]^{-1}\ Q_{pxa,bp}{}'\ [\mathrm{diag}(q_{p,bp})]^{-1}} \qquad (59)$$

The *fundamental analytical equation for the segment of domestic products based on the industry technology assumption (equation 58) may be solved even in the case that the production table is not square*. The solution of the system is:

$$\mathbf{q}_{p,bp,uD} = (\mathbf{I} - \mathbf{B}_{pxa,D}\,\mathbf{D}_{axp})^{-1}\mathbf{q}_{p,bp,uD}\,\mathbf{F}_{pxf,D}\;\mathbf{u} \qquad (60)$$

We obtain the correct results to estimate the *product-by-product SIOT of intermediate consumption matrix for domestic products based on the industry technology assumption*:

$$\mathbf{U}_{A,pxp,bp,D} = \mathbf{B}_{pxa,D}\,\mathbf{D}_{axp}\,\mathrm{diag}(\mathbf{q}_{p,bp,uD}) =$$
$$\mathbf{U}_{pxa,bp,D}\,[\mathrm{diag}(\mathbf{q}_{a,bp})]^{-1}\,\mathbf{Q}_{pxa,bp}{'}\,[\mathrm{diag}(\mathbf{q}_{p,bp})]^{-1}\,\mathrm{diag}(\mathbf{q}_{p,bp,uD}) \qquad (61)$$

Consequently, the intermediate use matrix is symmetric since in the rows are used the same units than in the columns (products in rows and columns).

The value added matrix by products based on industry the technology assumption ($\mathbf{V}_{A,vxp}$)

Derived from the *fundamental analytical equation for the segment of domestic products based on the industry technology assumption* and the estimation of the product-by-product SIOT of technical coefficients *based on the industry technology assumption* we obtain:

$$\mathbf{q}_{p,bp,uD} = \mathbf{A}_{A,pxp,bp,D}\,\mathbf{q}_{p,bp,uD} + \mathbf{F}_{pxf,D}\;\mathbf{u} = \mathbf{U}_{A,pxp,bp,D}\,\mathbf{u} + \mathbf{F}_{pxf,D}\,\mathbf{u} \qquad (62)$$

Thus, it is deducted that the sum for the rows of the *product-by-product SIOT of intermediate consumption for the segment of domestic products based on the industry technology assumption* is equal to the post-multiplication of the *product-by-product*

SIOT of technical coefficients for the segment of domestic products with industry technology **by the** ***total use vector of domestic products at basic prices with trade and transportation margins integrated*****:**

$$\mathbf{A}_{A,pxp,bp,D}\ \mathbf{q}_{p,bp,uD} = \mathbf{U}_{A,pxp,bp,D}\ \mathbf{u} \qquad (63)$$

Considering that:

$$\mathbf{u} = \mathrm{diag}(\mathbf{q}_{p,bp,uD})^{-1}\ \mathbf{q}_{p,bp,uD} \qquad (64)$$

Then:

$$\mathbf{A}_{A,pxp,bp,D}\ \mathbf{q}_{p,bp,uD} = \mathbf{U}_{A,pxp,D}\ \mathrm{diag}(\mathbf{q}_{p,bp,uD})^{-1}\ \mathbf{q}_{p,bp,uD} \qquad (65)$$

In other words:

$$\mathbf{B}_{pxa,D}\ \mathbf{D}_{pxa}\ \mathbf{q}_{p,bp,uD} = \mathbf{U}_{A,pxp,bp,D}\ \mathrm{diag}(\mathbf{q}_{p,bp,uD})^{-1}\ \mathbf{q}_{p,bp,uD} \qquad (66)$$

Reason why:

$$\mathbf{B}_{pxa,D}\ \mathbf{D}_{pxa}\ \mathbf{q}_{p,bp,uD}\ \mathbf{q}_{p,bp,uD}{}' = \mathbf{U}_{A,pxp,bp,D}\ \mathrm{diag}(\mathbf{q}_{p,bp})^{-1}\ \mathbf{q}_{p,bp,uD}\ \mathbf{q}_{p,bp,uD}{}' \qquad (67)$$

And:

$$\mathbf{B}_{pxa,D}\ \mathbf{D}_{pxa}\ (\mathbf{q}_{p,bp,uD}\ \mathbf{q}_{p,bp,uD}{}')\ (\mathbf{q}_{p,bp,uD}\ \mathbf{q}_{p,bp,uD}{}')^{-1} = \mathbf{U}_{A,pxp,bp,D}\ \mathrm{diag}(\mathbf{q}_{p,bp,uD})^{-1} \qquad (68)$$

Then:

$$\mathbf{B}_{pxa,D}\,\mathbf{D}_{pxa} = \mathbf{U}_{A,pxp,bp,D}\ \mathrm{diag}(\mathbf{q}_{p,bp,uD})^{-1} \qquad (69)$$

Consequently:

$$\mathbf{U}_{A,pxp,bp,D} = \mathbf{B}_{pxa,D}\,\mathbf{D}_{axp}\ \mathrm{diag}(\mathbf{q}_{p,bp,uD}) = \mathbf{U}_{pxa,bp,D}\,[\mathrm{diag}(\mathbf{q}_{a,bp})]^{-1}\,\mathbf{Q}_{pxa,bp}'\,[\mathrm{diag}(\mathbf{q}_{p,bp})]^{-1}\ \mathrm{diag}(\mathbf{q}_{p,bp,uD}) = \mathbf{U}_{pxa,bp,D}\,[\mathrm{diag}(\mathbf{q}_{a,bp})]^{-1}\,\mathbf{Q}_{pxa,bp}' \qquad (70)$$

In other words:

$$\mathbf{U}_{A,pxp,bp,D} = \mathbf{U}_{pxa,bp,D}\,[\mathrm{diag}(\mathbf{q}_{a,bp})]^{-1}\,\mathbf{D}_{axp}\ \mathrm{diag}(\mathbf{q}_{p,bp,uD}) \qquad (71)$$

With it we obtain the transformation matrix for the columns of the value added matrix of vxa dimensions:

$$\mathrm{diag}(\mathbf{q}_{a,bp})^{-1}\,\mathbf{D}_{axp}\ \mathrm{diag}(\mathbf{q}_{p,bp,uD}) \qquad (72)$$

In other words, the *value added matrix based on industry technology assumption* with vxp dimensions would be:

$$\mathbf{V}_{A,vxp} = \mathbf{V}_{vxa}[(\mathrm{diag}(\mathbf{q}_{a,bp})]^{-1}\,\mathbf{D}_{axp}\ \mathrm{diag}(\mathbf{q}_{p,bp,uD}) = \mathbf{V}_{vxa}[(\mathrm{diag}(\mathbf{q}_{a,bp})]^{-1}\,\mathbf{Q}_{pxa,bp}'\,[\mathrm{diag}(\mathbf{q}_{p,bp})]^{-1}\ \mathrm{diag}(\mathbf{q}_{p,bp,uD}) = \mathbf{V}_{vxa}[(\mathrm{diag}(\mathbf{q}_{a,bp})]^{-1}\,\mathbf{Q}_{pxa,bp}' \qquad (73)$$

The *value added vector based on the industry technology assumption* would be:

$$\boxed{v_{A,p,bp}{}' = u'\,V_{A,vxp} = u'\,V_{vxa}\,[diag(q_{a,bp})]^{-1} D_{axp}\ diag(q_{p,bp,uD})} \quad (74)$$

3.1.5 Industry-by-industry SIOT of technical coefficients for the segment of domestic products based on the fixed product sales structure (market quotas) assumption.

Substituting 18 into 2 we obtain:

$$q_{a,bp} = D_{axp}\ q_{p,bp} = D_{axp}\ q_{p,bp,uD} \quad (75)$$

In other words:

$$q_{p,bp,uD} = (D_{axp})^{-1}\ q_{a,bp} \quad (76)$$

Then, replacing it in the *fundamental input-output analysis* *asymmetric analytical equation for the segment of domestic products* derived directly from the SUTbp:

$$(D_{axp})^{-1}\ q_{a,bp} = U_{pxa,bp,D}\ u + F_{pxf,D}\ u = B_{pxa,D}\ q_{a,bp} + F_{pxf,D}\ u \quad (77)$$

In this way we obtain the *fundamental symmetric input-output analytical equation for the segment of domestic products based on fixed product sales structure (market quotas) assumption* mathematically derived from SUTbp:

$$\boxed{q_{a,bp} = D_{axp}\ B_{pxa,D}\ q_{a,bp} + D_{axp}\ F_{pxf,D}\ u} \quad (78)$$

This equation is a linear equation system on which the production vector of the industries is a linear function of itself $q_{a,bp} = f(q_{a,bp})$.

Thus, the symmetric technical coefficients matrix industry-by-industry for the segment of domestic products based on fixed product sales structure (market quotas) assumption, is:

$$\mathbf{A_{FPS,axa,bp,D} = D_{axp}\ \ B_{pxa,D} = Q_{pxa,bp}'\ [diag(q_{p,bp})]^{-1}\ U_{pxa,bp,D}[diag(q_{a,bp})]^{-1}} \qquad \textbf{(79)}$$

Thus, the ***<u>fundamental symmetric input-output equation for the segment of domestic products based on the fixed sales structure of the products (market quotas)</u>*** **may be solved even when the production matrix is not square. The solution of the system is:**

$$\boxed{\mathbf{q_{a,bp} = (I- D_{axp}\ B_{pxa,D})^{-1}\ D_{axp}\ F_{pxf,D}\ u}} \qquad \textbf{(80)}$$

We obtain the correct results to estimate the ***<u>symmetric intermediate consumption matrix industry-by-industry for the segment of domestic products based on the fixed product sales structure (market quotas) assumption</u>*** **for the segment of domestic products:**

$$\mathbf{U_{FPS,axa,bp,D} = D_{axp}\ \ B_{pxa,D}\ diag(q_{a,bp}) = Q_{pxa,bp}'\ [diag(q_{p,bp})]^{-1}\ U_{pxa,bp,D}\ [diag(q_{a,bp})]^{-1}\ diag(q_{a,bp})} \qquad \textbf{(81)}$$

For that reason the total intermediate use matrix is symmetric since in the rows are used the same units than in the columns (industries in rows and columns).

The final demand matrix by industries based on fixed product sales structure (market quotas) assumption ($\mathbf{F_{FPS,a,bp,D}}$).

We observe that the products final demand has been transformed to industries according with the following equation:

$$\mathbf{D}_{axp}\ \mathbf{F}_{pxf,D}\ \mathbf{u} \qquad \textbf{(82)}$$

With it we obtain the transformation matrix for the columns of the value added matrix of vxa dimensions:

$$\mathbf{D}_{axp} \qquad \textbf{(83)}$$

In other words, the *value added matrix industry-by-industry based on fixed sales structure of industries* for the segment of domestic products of dimensions vxp would be:

$$\mathbf{F}_{FPS,axf,bp,D} = \mathbf{D}_{axp}\ \mathbf{F}_{pxf,D} = \mathbf{Q}_{pxa,bp}'\ [\mathrm{diag}(\mathbf{q}_{p,bp})]^{-1}\ \mathbf{F}_{pxf,D} \qquad \textbf{(84)}$$

Also, the *final demand value vector by based on fixed product sales structure (market quotas) assumption* for the segment of domestic products would be:

$$\boxed{\mathbf{f}_{FPS,a,bp,D}\ \ \mathbf{u} = \mathbf{D}_{axp}\ \mathbf{F}_{pxf,D}\ \mathbf{u}} \qquad \textbf{(85)}$$

3.2 The mathematical methods for the segment of imported products.

3.2.1 Product-by-product SIOT of technical coefficients for the segment of imported products based on production or commodity technology assumption.

Substituting 16 into 8 we obtain:

The *fundamental asymmetric analytical equation* of the segment of imported products of the input-output analysis derived directly from SUTbp:

$$\boxed{\mathbf{q}_{p,cif,uM} = \mathbf{B}_{pxa,M}\, \mathbf{q}_{a,bp} + \mathbf{F}_{pxf,M}\, \mathbf{u}} \qquad \textbf{(86)}$$

The imported products vector is a linear function of the production of activities $\mathbf{q}_{p,cif,uM} = \mathbf{f}(\mathbf{q}_{a,bp})$. But, this equation has no solution because in general $\mathbf{q}_{p,cif,uM} \neq \mathbf{q}_{a,bp}$ even in the case that the number of products be equal to the number of industries. This analytical equation involves a use coefficients matrix by unit produced by industry ($\mathbf{B}_{pxa,M}$) asymmetric because in the rows are different units than those used in the columns (products in rows and activities in columns). Also because the intermediate use matrix of imported products, defined on the previous section (equation 16) is asymmetric too by the same reasons. It is equal to:

$$\mathbf{U}_{pxa,cif,M} = \mathbf{B}_{pxa,M}\ \mathrm{diag}(\mathbf{q}_{a,bp}) \qquad \textbf{(87)}$$

Solving $\mathbf{q}_{a,bp}$ from 17 we obtain two relations between $\mathbf{q}_{a,bp}$ and $\mathbf{q}_{p,bp,uD}$:

$$\mathbf{q}_{a,bp} = \mathbf{f}(\mathbf{q}_{p,bp,uD}) \qquad \textbf{(88)}$$

$$\mathbf{q}_{a,bp} = (\mathbf{D}_{pxa})^{-1}\, \mathbf{q}_{p,bp,uD} \qquad \textbf{(89)}$$

Also:

$$\mathbf{q_{p,bp,uD} = f(q_{a,bp})} \qquad \textbf{(90)}$$

$$\mathbf{q_{p,bp,uD} = D_{pxa} \; q_{a,bp}} \qquad \textbf{(91)}$$

And substituting 89 in the _asymmetric analytical equation of the segment of imported goods_ in order to obtain the _fundamental symmetric input-output analytical equation of imported goods based on the production or commodity technology assumption_ mathematically derived from SUTbp:

$$\mathbf{q_{p,cif,uM} = B_{pxa,M} \, (D_{pxa})^{-1} \, q_{p,bp,uD} + F_{pxf,M} \, u} \qquad \textbf{(92)}$$

With this equation is obtained $\mathbf{q_{p,cif,uM}}$ once $\mathbf{q_{p,bp,uD}}$ is known.

Thus, we obtain the _fundamental symmetric technical coefficients matrix product-by-product of imported goods based on the production or commodity technology assumption_ mathematically derived from SUTbp:

$$\boxed{\mathbf{A_{P,pxp,cif,M} = B_{pxa,M} \, (D_{pxa})^{-1} =}}$$
$$\boxed{\mathbf{U_{pxa,bp,M} \; [diag(q_{a,bp})]^{-1} diag(q_{a,bp})] Q_{pxa,bp}^{-1}}} \qquad \textbf{(93)}$$

Requires $\mathbf{Q_{pxa,pb}}$ squared and:

$$U_{P,pp,cif,M} = B_{pxa,M}\,(D_{pxa})^{-1}\,diag(q_{p,bp,uD}) =$$
$$Q_{pxa,bp}[diag(q_{a,bp})]^{-1}U_{pxa,bp,M}[diag(q_{a,bp})]^{-1}[diag(q_{a,bp})]Q_{pxa,bp}{}^{-1}diag(q_{p,bp,uD})$$
$$=U_{pxa,cif,M}\;\;Q_{pxa,bp}{}^{-1}\;\;diag(q_{p,bp,uD}) \tag{94}$$

3.2.2 Product-by-product SIOT of technical coefficients for the segment of imported products based on the industry technology assumption.

Substituting 16 into 8:

$$q_{p,cif,uM} = B_{pxa,M}\;q_{a,bp} + F_{pxf,M}\;u \tag{95}$$

In 2 is substituted 18 and we have:

$$q_{a,bp} = D_{axp}\;q_{p,bp,uD} \tag{96}$$

With it we obtain a relation between $q_{a,bp}$ and $q_{p,bp,uD}$ its result is substituted by $q_{a,bp}$ in the analytical equation 85 to obtain the *fundamental symmetric input-output analytical equation for imported products based on the industry technology assumption* mathematically derived from SUTbp:

$$q_{p,cif,uM} = B_{pxa,M}\;D_{axp}\;q_{p,bp,uD} + F_{pxf,M}\;u \tag{97}$$

Finally, we obtain the symmetric technical coefficients matrix product-by-product of imported goods based on industry technology assumption:

$$\boxed{A_{A,pxp,cif,M} = B_{pxa,M}\;D_{axp} =}$$
$$\boxed{U_{pxa,bp,M}\;\;[diag(q_{a,bp})]^{-1}\;Q_{pxa,bp}{}'\;[diag(q_{p,bp})]^{-1}} \tag{98}$$

It does not requiere that the production matrix $\mathbf{Q}_{pxa,pb}$ be squared and:

$$\mathbf{U}_{A,pp,cif,M} = \mathbf{B}_{pxa,M}\,\mathbf{D}_{axp}\,\mathrm{diag}(\mathbf{q}_{p,bp,uD}) =$$
$$\mathbf{U}_{pxa,bp,M}\,\left[\mathrm{diag}(\mathbf{q}_{a,bp})\right]^{-1}\mathbf{Q}_{pxa,bp}{}'\,\left[\mathrm{diag}(\mathbf{q}_{p,bp})\right]^{-1}\mathrm{diag}(\mathbf{q}_{p,bp,uD}) =$$
$$\mathbf{U}_{pxa,bp,M}\,\left[\mathrm{diag}(\mathbf{q}_{a,bp})\right]^{-1}\mathbf{Q}_{pxa,bp}{}' \qquad (99)$$

3.2.3 Industry-by-industry SIOT of technical coefficients for the segment of domestic products based on fixed product sales structures assumption (market quotas).

The derivation of the *__fundamental symmetric input-output analytical equation based on the fixed sales structure of products (market quotas)__* mathematically derived from SUTbp for the segment of domestic products, rests basically in the definition of the market quotas matrix, through which an equation is obtained that relates the production vector of industries and the vector of production of the products and, by adding, it relates the vector of production of the products with the vector of production of the industries; the equations 75 and 76.

For the imports case it is not feasible to use these relations; thus, it is not feasible to derive mathematically – for the imported goods segment – the *__fundamental symmetric input-output analytical equation based on fixed product sales structures assumption (market quotas)__* from the SUTbp.

3.2.4 Industry-by-industry SIOT of technical coefficients for the segment of imported products based on fixed industry sales structure assumption.

The derivation of the *fundamental symmetric input-output analytical equation based on fixed industry sales structure assumption* mathematically derived from SUTbp for the segment of domestic products, rests basically on the definition of the matrix of market shares or market quotas (matrix of output coefficients for products by industry):, through which is obtained an equation that relates the vector of production of the industries and the vector of production of the products and in addition relates the vector of production of the products with the vector of production of the industries; equations 89 and 90.

For the case of imports these relations do not exist; thus, it is not feasible to derive mathematically for the segment of imported goods the *fundamental symmetric input-output analytical equation based on fixed industry sales structure assumption* from the SUTbp.

Conclusions

The input-output theoretical approach, especially that related with physical units input-output tables is a complex subject, as noted in the following quote:

"…an I/O coefficient table may be compiled directly, using engineering information, without first creating an input-output table in monetary values. To do so, one may ask detailed technical information on all production processes operating in an economy. If a product is produced by more than one process, an average input structure of those production processes would be created by weighing input columns with output values. As weights change, the average input structures would have to be frequently updated. With additional information on gross output, value added and final demand, it is possible to recreate a flow table using the I/O coefficient matrix and output prices and thus check the accuracy and compatibility of the engineering information with the data on value added and final demand, in the final balancing of the table. This construction of I/O tables in physical terms is not simple, since enterprises may be reluctant to reveal technical details of their production processes. Thus technical data may have to be collected through special surveys, independently of censuses, and this may be very expensive. This is why the compilation of I/O tables has relied mainly on data provided by censuses of establishments and special surveys. This information could be supplemented by engineering-type data but this is rarely done.…" (United Nations, 2000; 4.2).

Due to these drawbacks and to current practices, especially by cost constraints, actual research has considered SIOT in physical units basically to conceptualize the theoretical underlying model and trying to justify the efforts for the estimation of the symmetric matrix from the supply-use tables that contain secondary products. All this, in order to obtain elements that could sustain the choice among different methods to estimate the SIOT as an analytical tool based on the theoretical input-output model.

For the selection among several functional methods to comply with the theoretical input-output model characteristics, it is necessary to emphasize the importance of homogeneity in the production functions between the elements from the different columns of the symmetric matrix and, consequently, between their costs′ structure. It

is a *condicio sine qua non* in order to obtain higher precision in the input-output analysis. Thus, the international patterns define the ideal situation as:

"The ideal situation for a symmetric I/O table is to have data that describe the input structure of every type of activity producing a single product in the economy. With such an ideal situation, the I/O table is almost symmetric and homogeneity in production function in the table is guaranteed, except for the cases of byproducts or joint products that are linked technologically in a production activity. If homogeneity is not guaranteed, a distortion in analysis; particularly when total effects are calculated by the use of the Leontief inverse may result." (United Nations, 2000; 4.3).

Three conclusions result from the previous analysis which are central for the target of this research.

First, the main conclusion is that most of all the redefinition method should be employed before applying any mathematical method.

This work can be easily realized if the productive units that provide the information be ready to supply it in an *ad hoc* form for these goals. It is the wish of the compilators and the hope of the mentioned international norms, as expressed in the next quote:

"Partitioning enterprises into units of homogeneous production is an important task in the collection of production data and, as a consequence, the compilation of the symmetric I/O table. The best way to get inputs and outputs from the units of homogeneous production in an enterprise is to ask it to do so with clear instructions from data collectors so that there will be no need to use mechanical methods to separate them afterwards." (United Nations, 2000; 4.13).

Nevertheless:

"In contrast to the ideal situation, the United Nations (2000) use and supply tables as an integrated framework for production statistics are designed to serve as the best statistical tool to compile national accounts aggregates and provide information for the compilation of the symmetric I/O table." (United Nations, 2000; 4.4).

Therefore, the redefinition method should be fully used before applying the mathematical methods. As it is written on the United Nations input-output Manual:

"Given that data are collected on an establishment basis and consist of secondary products which are not by-products or joint products resulting from production processes, it is still beneficial to collect input data independently for the secondary products of the same kind and use this information to estimate the inputs associated with the secondary products and transfer them out."(United Nations, 2000; 4.24).

Second, basically it has been analyzed the depth of the four mathematical methods used *as last resort* for the estimation of the SIOT and coverage was stretched for the foreign sector. Briefly:

Scheme 5 Symmetric matrices of technical coefficients estimated with different methodologies

	Domestic		Imported	
Product-by-product SIOT of technical coefficients for the segment of domestic and imported products based on production or commodity technology assumption.	$A_{P,pp,bp,D}$	$B_{pxa,D}(D_{pxa})^{-1}$	$A_{P,pp,cif,M}$	$B_{pxa,M}(D_{pxa})^{-1}$
Industry-by-industry SIOT of technical coefficients for the segment of domestic products based on fixed industry sales structure assumption.	$A_{FIS,aa,bp,D}$	$(D_{pxa})^{-1}B_{pxa,D}$		
Product-by-product SIOT of technical coefficients for the segment of domestic and imported products based on industry technology assumption.	$A_{A,pxp,bp,D}$	$B_{pxa,D}D_{axp}$	$A_{A,pp,cif,M}$	$B_{pxa,M}D_{axp}$
Industry-by-industry SIOT of technical coefficients for the segment of domestic products based on fixed product sales structures assumption (market quotas).	$A_{FPS,aa,bp,D}$	$D_{axp}B_{pxa,D}$		

The second conclusion is: this mathematical demonstration implies that the equations proposed on the Manual (United Nations, 2000) are adequate to estimate

the product-by-product SIOT for the total inputs used in the production process. This is because from Section 2, it can be inferred the following:

$$\mathbf{B}_{pxa,T} = \mathbf{B}_{pxa,M} + \mathbf{B}_{pxa,D}. \qquad (100)$$

So that:

Scheme 6 Symmetric matrices of total technical coefficients estimated with different methodologies		
Product-by-product SIOT of technical coefficients for total products (domestic and imported) based on production or commodity technology assumption.	$\mathbf{A}_{P,pp,bp,T}$	$\mathbf{B}_{pxa,T}\,(\mathbf{D}_{pxa})^{-1}$
Product-by-product SIOT of technical coefficients for total products (domestic and imported) based on industry technology assumption.	$\mathbf{A}_{A,pxp,bp,T}$	$\mathbf{B}_{pxa\,,T}\,\mathbf{D}_{axp}$

However, such mathematical demonstration does not imply that it has been mathematically demonstrated or intuitively or imperatively indicated in the Manual mentioned, causing and still creating a great confusion for most of the compilers in several countries. Furthermore, if the Manual instructions are followed step by step, a huge vacuum appears for the feasibility of the input-output analysis, since on said Manual one calculation procedure is referred only, from it the estimation of the symmetric imports matrix is unfortunately missing, it describes the CIF valued imports vector of the supply table and the CIF/FOB adjustment only¡ This represents a major limitation for the input-output analysis about the *in crescendo* foreign trade flows of the economic globalization. Meanwhile, from the equations developed and demonstrated in this research it is feasible to estimate such import matrices and support projects such as the mentioned by OECD, which are seriously handicaped by

the availability of methodologies that allow comprehensive international comparisons. See (Norihiko Yamano and Nadim Ahmad, 2006).

This second conclusion is also applied to calculate symmetric flows matrices estimated with different methodologies with the value added and final demand coefficients matrices associated with these, as shown in scheme 7.

Third, the product-by-product SIOT is adequate for the economic analysis, as noticed for her closeness to the theoretical model described at detail in the literature, especially in chapter I.2 of (Haro García, R. de J., 2006). We have analyzed two methods that mathematically combine SUTbp in order to develop the product-by-product SIOT. These methods are based on the industry technology or the product technology assumptions, the latter being the most adequate because results are acceptable from the Economics approach and because from the theoretical model is relatively easy to probe that it satisfies the criteria based on it (Pieter K. Jansen y Ten Raa Thijs, 1990) y (Ten Raa Thijs, D. Chakraborty y J. A. Small, 1984), as expressed in the following statements:

- **Material balance: Total output is equal to total intermediate consumption plus final demand;**
- **Financial balance: For every industry, the price equation will hold when applied to revenues and costs of producers;**

Scheme 7 Value added and final demand from SIOT estimated with different methodologies				
	Value Added		**Final Demand**	
Value added matrix based on production or commodity technology assumption.	$V_{P,vxp}$	$V_{vxa}\,(Q_{pxa,bp})^{-1}\,\mathrm{diag}(q_{p,bp})$	**As in the use table**	
Final demand matrix based on Fixed industry sales structures assumption.	**As in the use table**		$F_{FIS,axf,bp,D}\;u$	$(D_{pxa})^{-1}\,F_{pxf,D}\;u$
Value added matrix based on industry technology assumption.	$V_{A,vxp}$	$V_{vxa}[(\mathrm{diag}(q_{a,bp})^{-1}]D_{axp}\,\mathrm{diag}(q_{p,bp,uD})$	**As in the use table**	
Final demand matrix based on Fixed product sales structures assumption (market quotas).	**As in the use table**		$F_{FPS,axf,bp,D}$	$D_{axp}\,F_{pxf,D}$

Scheme 8 Symmetric flows matrices estimated with different methodologies				
	Domestic flows		**Imported flows**	
Product-by-product SIOT of flows for the segment of domestic and imported products based on production or commodity technology assumption.	$U_{P,pxp,bp,D}$	$B_{pxa,D}\,(D_{pxa})^{-1}\,diag(q_{p,bp,uD}) = U_{pxa,bp,D}\,Q_{pxa,bp}^{-1}\,diag(q_{p,bp,uD})$	$U_{P,pp,cif,M}$	$B_{pxa,M}(D_{pxa})^{-1}\,diag(q_{p,bp,uD}) = U_{pxa,cif,M}\,Q_{pxa,bp}^{-1}\,diag(q_{p,bp,uD})$
Industry-by-industry SIOT of domestic flows based on fixed industry sales structures assumption.	$U_{FIS,aa,bp,D}$	$(D_{pxa})^{-1}\,B_{pxa,D}\,diag(q_{a,bp}) = [diag(q_{a,bp})]\,Q_{pxa,bp}^{-1}\,U_{pxa,bp,D}$	**Not applicable**	
Product-by-product SIOT of flows for the segment of domestic and imported products based on industry technology assumption.	$U_{A,pxp,bp,D}$	$B_{pxa,D}\,D_{axp}\,diag(q_{p,bp,uD}) = U_{pxa,bp,D}\,[diag(q_{a,bp})^{-1}]\,Q_{pxa,bp}'$	$U_{A,pp,cif,M}$	$B_{pxa,M}\,D_{axp}\,diag(q_{p,bp,uD}) = U_{pxa,bp,M}\,[diag(q_{a,bp})^{-1}]\,Q_{pxa,bp}'$
Industry-by-industry SIOT of domestic flows based on fixed product sales structures assumption (market quotas).	$U_{FPS,aa,bp,D}$	$D_{axp}\,B_{pxa,D}\,diag(q_{a,bp}) = Q_{pxa,bp}'[diag(q_{p,bp})^{-1}]\,U_{pxa,bp,D}$	**Not applicable**	

- **Scale invariance: The derived symmetric coefficient matrix is also invariant to a scaling factor, i.e. if inputs and output of an industry in the original use and make matrices are increased by the same proportion, it is possible to derive the same coefficient matrix;**

- **Price invariance: If a new price base is applied to the data, the same derived symmetric input-output coefficient matrix A is obtained.**

About this it is necessary to recall an old but effective basic approach for the measuring of economic phenomena:

"In recent years…two different approaches to economic problems, the factual and the theoretical, have been brought much close together. In attempting to give quantitative expression to empirical constructs, such as the national income, it is now generally recognized that a theoretical basis is necessary and that *this basis should be the conscious concern of economists and not left in its practical aspect exclusively to business men, accountants and the Commissioners of Inland Revenue*. Equally is it clear that economic theory cannot usefully be left at the theoretical stage but requires to be tested and given quantitative expression by being brought into relation with observations. These lines of attack have resulted in very considerable efforts to bring into being both observations which are relevant to economic theories, and also theories, or formulation of theories, which are capable of being brought into relation with observations" (Stone Richard, 1951). Italics are from the author of this research.

However, the question continues unsolved. However, the OECD herself holds a position favoring industry-by-industry SIOT, as shown in the next paragraph:

"… in practice, the conversion to industry-by-industry tables best preserves the inter-industrial economic relationships that users are interested in. Most importantly it means that value-added and its components by industry are exactly the same as shown in supply-use tables (and so remain consistent with real data returns). This is not the case of course for commodity-by-commodity tables where, whatever the conversion methods used, value-added and its components are affected". (Norihiko Yamano and Nadim Ahmad, 2006)

Recommendations

The method of redefinition and the mathematical transformations

As far as serious aggregation problems remain in the input-output analysis in order to obtain homogeneous production units and because mathematical methods do not lead to the desired homogeneous production units, the most appropriate procedure is, if possible, to define in an operative form the most pure establishment, linking each product to the industry where it belongs until it is integrated to homogeneous production units, only.

It is necessary to design information collection procedures (i. e. surveys) in which business accountants be asked to use analytical methods designed by the engineers in charge of production processes and assign costs for each product produced by an establishment in surveys questionnaires. On big establishments, this is more feasible because cost accounting is a common practice compared to medium and small ones.

In doing this, most secondary products will be eliminated from SUT. Mathematical methods which are basically mechanical will be used only as a last resort. (United Nations, 2000; 4.4).

But this is not usual in most countries yet, especially in emerging and economic and socially backward countries. Therefore, I insist again that the redefinition method has to be used before applying mathematical methods. Experts from each industry know their production chain, thus they are able to determine directly which inputs will be assigned to secondary products. It is about a method like the mathematical method based on the production technology assumption. However, in this case a group of

experts from each industry will be directly involved in the decision-making about which are the independent inputs that are assigned to secondary products.

The production of secondary products will be held anyway, which will have to be treated mathematically using the production technology assumption in order to approach the homogeneous production units. However, it is also necessary to solve the problem of negative numbers; Therefore, it is necessary to recall that the technical coefficients obtained from this method could be negative in the following cases (United Nations Organization, 2000; 4.69):

- **The output of the secondary product in the make matrix (the supply table) is misclassified;**
- **The secondary product is not exactly the same as the product produced as a primary product elsewhere; it requires less inputs than assumed;**
- **There are errors in basic data.**

Some implications about the distinction between imports and domestic use tables

As it was demonstrated by the development of this research, activity-by-activity symmetric technical coefficients tables estimated with the fixed industry sales structures or the fixed product sales structures (fixed market quotas) assumptions, is not feasible to estimate them with mathematical methods when goods and services are imported.

This mathematical demonstration is crucial due to the generalized practice in many developed countries that make these estimations based on the fixed market quotes assumption. The following paragraph is a clear sample about this issue:

> **So far it has implicitly been assumed that domestic output and imports of products belonging to the same product group have been lumped together in the SUT (and in the SIOT). However, analytical users are often interested in a distinction between effects on the domestic economy and on imports. In order to facilitate this, the elements in the SIOT must be broken down according to domestic and import origin. This breakdown should take place at the most detailed product level possible, and therefore preferable in the rectangular SUT. No matter what type of data is available or which method is used, this split obviously means that market shares (and in practice market share assumptions) will be build into the core part of the input-output analytical framework. When the split between supply from domestic industries and imports is made on the assumption of constant import ratios along the row (except for exports where actual information is usually available) this market share assumption is identical to the one used in the construction of the industry-by-industry SIOT on the assumption of fixed product sales structures. It can be demonstrated that the resulting “domestic” and “import” matrices are very sensitive to the level of product aggregation at which the split takes place. (This can be seen as an analogy to aggregation error suffered when compiling the SIOT from the square rather than the rectangular SUT). Thus the import matrices resulting from a split at a detailed SUT level will better reflect reality than those obtained if the split were carried out at the level of the square SUT or directly in the SIOT. The split between domestic output and imports must usually be based on the market share assumption, and will probably affect the final outcome for the domestic SIOT more than the choice of a "technology" assumption. Whereas the application of a product technology assumption to transform the “domestic” SUT into a SIOT would seem to lack logic it is quite straightforward to use the assumption of the fixed product sales structure to obtain the domestic industry-by-industry SIOT . <u>The same procedure can be used to transform the (rectangular) import supply matrix into an industry-by-industry format</u>. (Thage, Bent, 2005). (The underlining is by the author of this paper).**

On the contrary, when it has been decided to estimate the SIOT activity-by-activity, the mathematical demonstrations developed in this research have indicated without any doubts about the inappropriate use of mathematical methods for imports calculation; the conducive thing here is a planning work about the SUTbp and SIOT in such a manner that the imports estimation for SIOT activity-by-activity be by aggregation, which implies a correspondence without confusion between the product and activities classifications. It can be done also using information systems that have

built-in this question and its solution when passing from SUTbp to the symmetric imports table in the case of all products and tariff fractions, even in this case some confusion degree to the information users will still be done that would not allow to calculate it by the aggregation of products.

This major problem is still waiting for a solution from the Statistics Commission of the United Nations. Because there is not a working correspondence item by item between the highest aggregation level of the ISIC (United Nations Organization, 1990) and the highest disaggregated level of the CPC (United Nations Organization, 2002). This drawback has obligated several countries (i.e. Canada, USA, Japan, European Union, among others), to construct their own product classifications at the higher trade-off by loosing the international comparatibility, a serious problem given the economic globalization and the depth of the interdependence among countries, facing the urgent necessity for structured works that help to understand the new patterns and trends of foreign trade at a detailed level.

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